PHY 452/562
Lasers and Modern Optics

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Lecture 9: Quantum Mechanics IV

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- Wave-particle duality.
- Uncertainty principle.
- Schroedinger’s equation

Wave-particle duality

De Broglie's viewpoint was the assumption that momentum-wavelength relation is true for both photons and massive particles.

The De Broglie wave equations are:

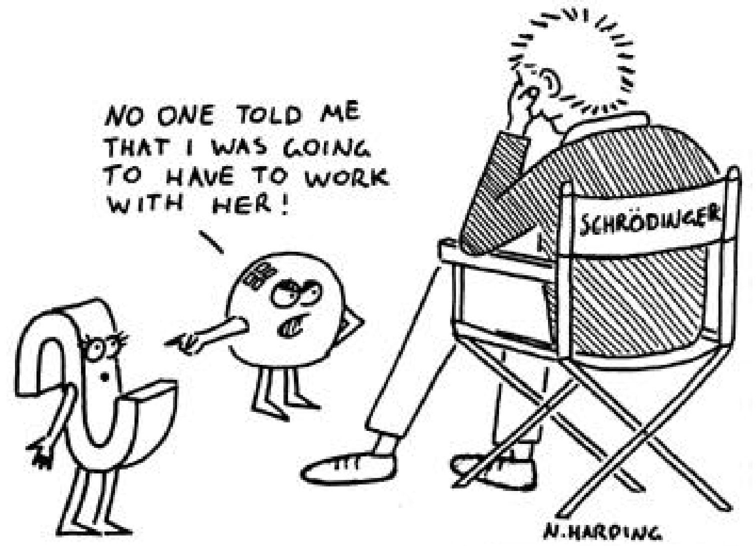
$$\lambda = \frac{h}{p} = \frac{h}{mV}$$

$$E = mc^2 = h\nu$$

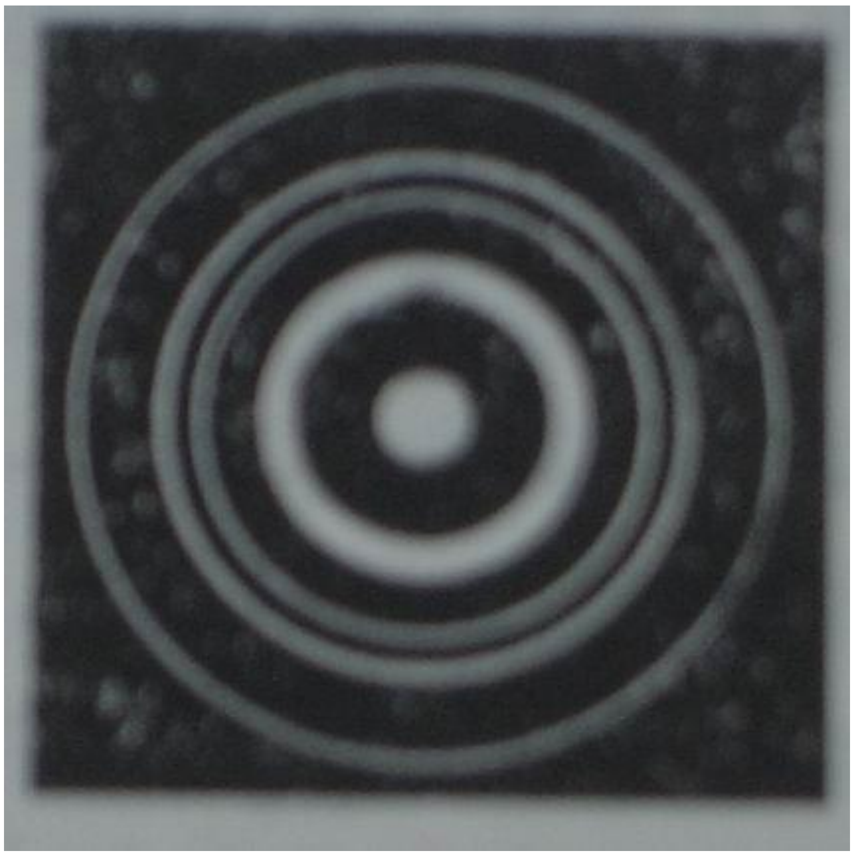
-The classical concept of an electron is a point particle of definite mass and charge.

-De Broglie argued that the wavelength of the wave associated with an electron might be so small that it had not been previously noticed.

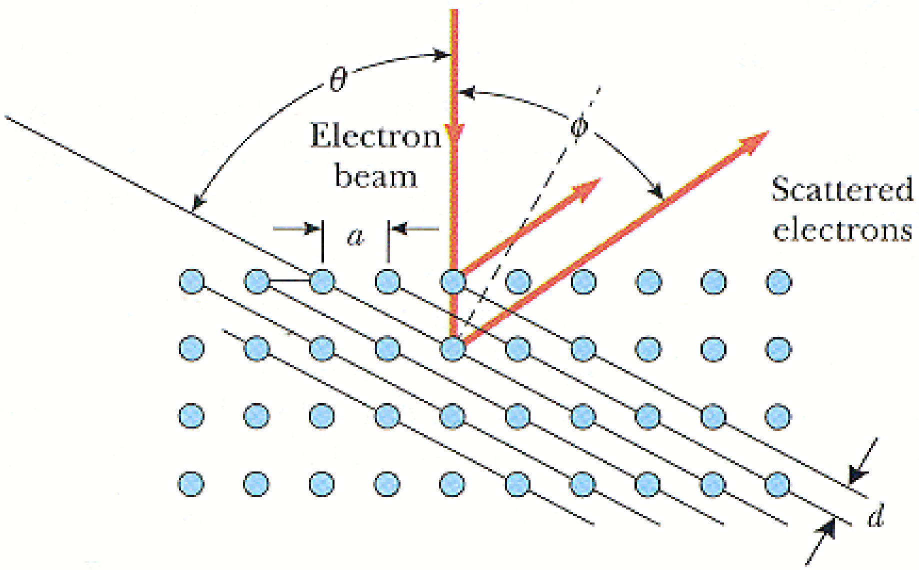
-If we wish to prove that an electron has a wave nature, we must perform an experiment in which electrons behave as waves.



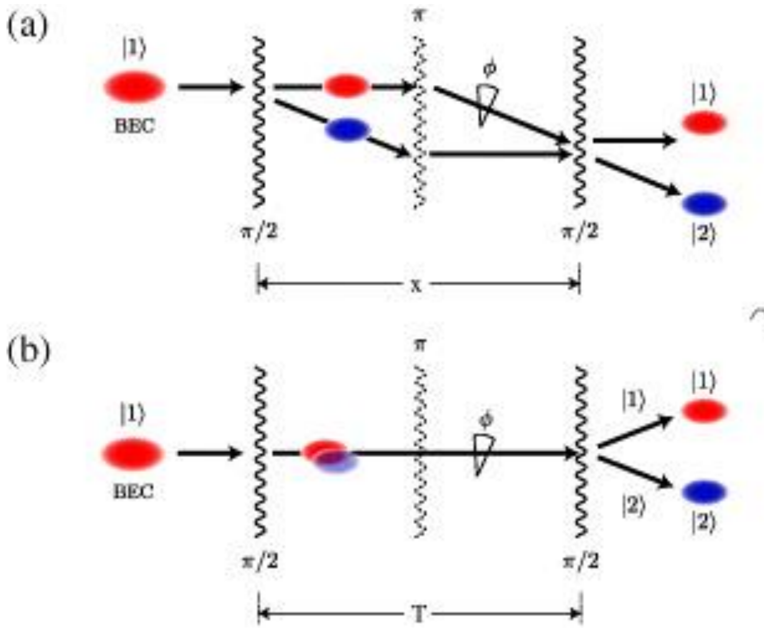
For example, de Broglie wavelength for an electron whose kinetic energy is 600 eV is 0.0501nm. The de Broglie wavelength for a golf ball of mass 45g traveling at 40m/s is: 3.68×10^{-34} m.

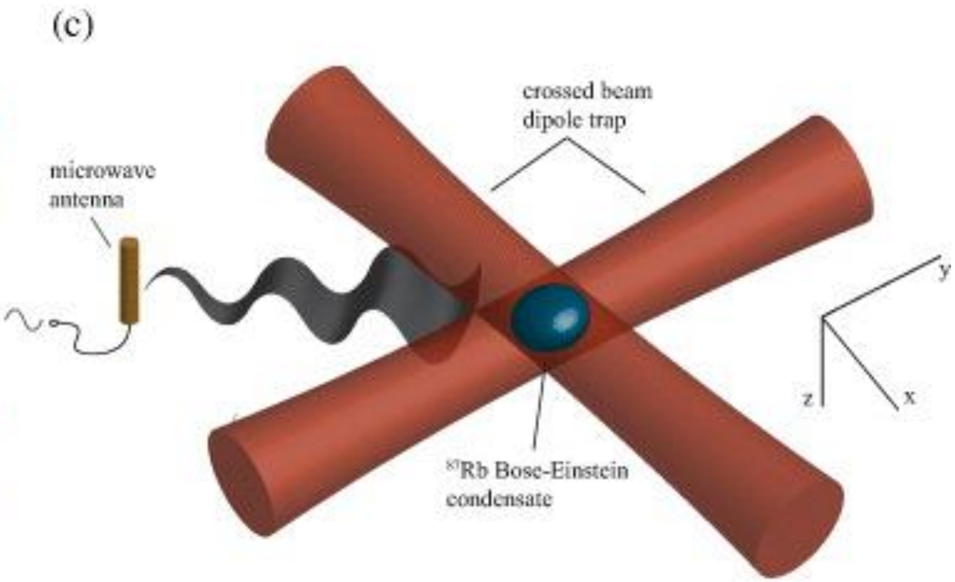


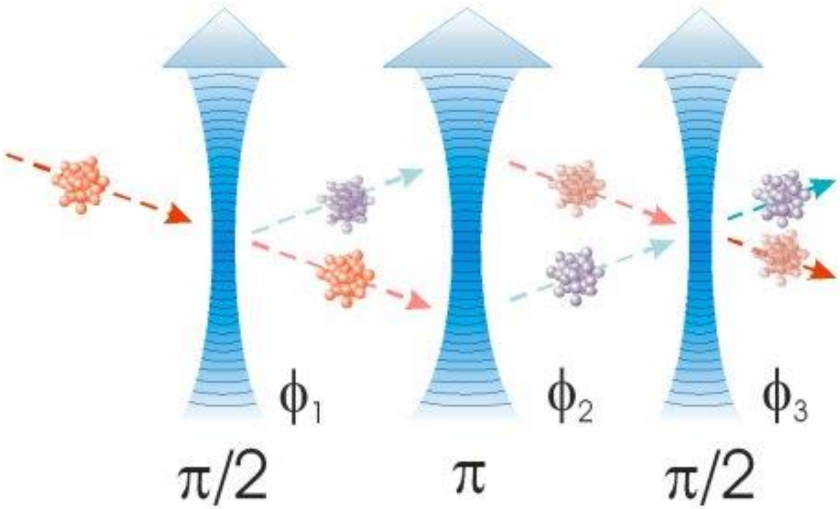
- Davisson–Germer experiment

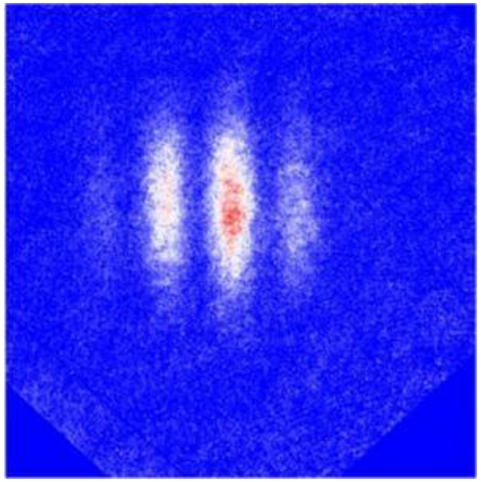


- Atom interferometry







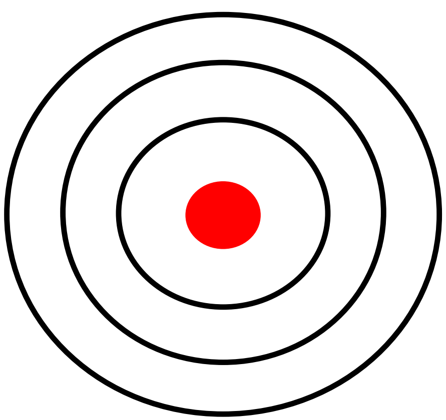


Uncertainty principle

- In the last lecture, we used Bohr's model of the atom to describe atomic behavior.

- Unfortunately, Bohr's mathematical interpretation is not correct when an atom has more than 1 electron.

- This failure is due to violation of the **Uncertainty principle**.



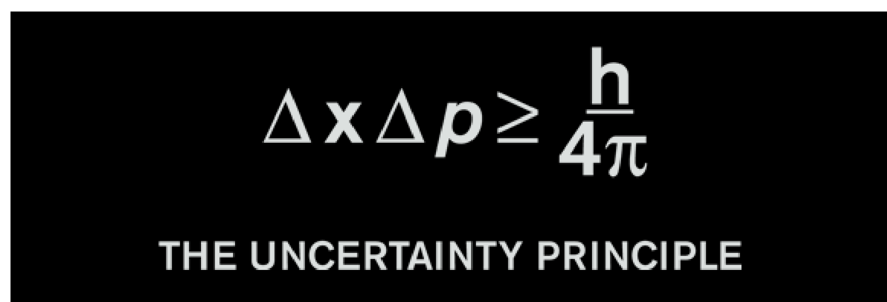
$$\left.\begin{aligned} &2\pi r = n\lambda \qquad n = 1, 2, 3\ldots \\ &E_n = \frac{-2.1799\ x\ 10^{-18}\ J}{n^2} \end{aligned}\right\} \text{Bohr's model}$$

- The uncertainty principle is a cornerstone of quantum theory.

- It asserts that:

  "You can NOT measure accurately both the position and momentum of an electron simultaneously, and this uncertainty is a fundamental property of the act of measurement itself"

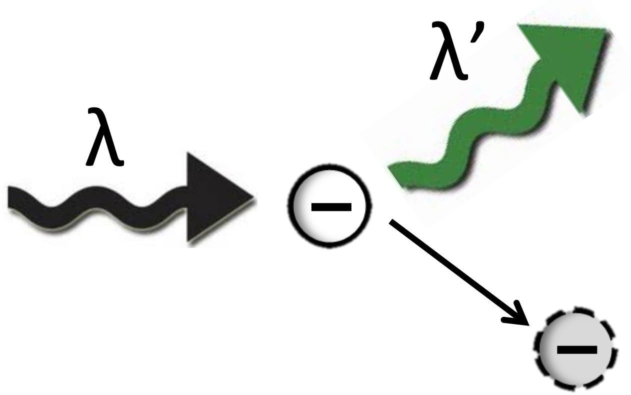
- This limitation is a direct consequence of the **wave-nature** of electrons



- If you wish to locate the electron within a distance Δx, we must use a photon with a wavelength which is equal to or less than Δx

- When the electron and photon interact, there is a change in momentum of the electron due to collision with the photon.

- **Thus, the act of measuring the position results in a change in its momentum.**



- If the minimum uncertainty (precision) in x is defined as Δx, and that of p is Δp:

$$(\Delta x)(\Delta p) \geq \frac{h}{4\pi}$$

- This indicates the **limit of the accuracy** of trying to *simultaneously* determine both an electron's position and momentum

- This uncertainty is **very significant** when applied to atomic and subatomic particles

- An electron is moving at a speed of 5.0×10^{6} m/s. You want to measure the position of an electron **within 5×10^{-11} m**. Estimate the uncertainty in the momentum, then the velocity.

$$(\Delta x)(\Delta p) \geq \frac{h}{4\pi}$$

$$(\Delta p) \geq \frac{h}{4\pi(\Delta x)}$$

$$(\Delta p) \geq \frac{(6.626\ x\ 10^{-34} Js)}{4\pi(5\ x\ 10^{-11}\ m)}$$

$$\boxed{(\Delta p) \geq 1\ x\ 10^{-24}\ kg\ m\ s^{-1}}$$

We assume the mass of the electron to be exact (m= 9.109 x 10^{-31} kg), so the uncertainty is only in the velocity

$$(\Delta p) = m(\Delta \bar{V})$$

$$m(\Delta \bar{V}) \geq 1\ x\ 10^{-24}\ kg\ m\ s^{-1}$$

Very large uncertainty (20% of actual speed)

$$\boxed{(\Delta \bar{V}) \geq 1\ x\ 10^{6}\ ms^{-1}}$$

- Bohr's model conflicts with the uncertainty principle because if the electron is set within a confined orbit, you know both its momentum and position at a given moment.