PHY 452/562
Lasers and Modern Optics

Prof. Eden Figueroa

Lecture 9: Quantum Mechanics IV
- Wave-particle duality.
- Uncertainty principle.
- Schroedinger’s equation
Wave-particle duality
De Broglie’s viewpoint was the assumption that momentum-wavelength relation is true for both photons and massive particles.

The De Broglie wave equations are:

\[ \lambda = \frac{h}{p} = \frac{h}{mV} \]

\[ E = mc^2 = h\nu \]
-The classical concept of an electron is a point particle of definite mass and charge.

-De Broglie argued that the wavelength of the wave associated with an electron might be so small that it had not been previously noticed.

-If we wish to prove that an electron has a wave nature, we must perform an experiment in which electrons behave as waves.
For example, de Broglie wavelength for an electron whose kinetic energy is 600 eV is 0.0501 nm. The de Broglie wavelength for a golf ball of mass 45g traveling at 40m/s is: $3.68 \times 10^{-34}$ m.

- Davisson–Germer experiment
- Atom interferometry
Uncertainty principle
• In the last lecture, we used Bohr’s model of the atom to describe atomic behavior.

• Unfortunately, Bohr’s mathematical interpretation is not correct when an atom has more than 1 electron.

• This failure is due to violation of the **Uncertainty principle**.

\[ 2\pi r = n\lambda \quad n = 1, 2, 3... \]

\[ E_n = \frac{-2.1799 \times 10^{-18}}{n^2} \quad \text{J} \]

Bohr’s model
• The uncertainty principle is a cornerstone of quantum theory.

• It asserts that:
  "You can NOT measure accurately both the position and momentum of an electron simultaneously, and this uncertainty is a fundamental property of the act of measurement itself"

• This limitation is a direct consequence of the wave-nature of electrons

\[ \Delta x \Delta p \geq \frac{\hbar}{4\pi} \]

THE UNCERTAINTY PRINCIPLE
• If you wish to locate the electron within a distance $\Delta x$, we must use a photon with a wavelength which is equal to or less than $\Delta x$.

• When the electron and photon interact, there is a change in momentum of the electron due to collision with the photon.

• Thus, the act of measuring the position results in a change in its momentum.
• If the minimum uncertainty (precision) in $x$ is defined as $\Delta x$, and that of $p$ is $\Delta p$:

\[
(\Delta x)(\Delta p) \geq \frac{h}{4\pi}
\]

• This indicates the **limit of the accuracy** of trying to simultaneously determine both an electron’s position and momentum

• This uncertainty is **very significant** when applied to atomic and subatomic particles
An electron is moving at a speed of $5.0 \times 10^6$ m/s. You want to measure the position of an electron \textbf{within $5 \times 10^{-11}$ m}. Estimate the uncertainty in the momentum, then the velocity.

\[(\Delta x)(\Delta p) \geq \frac{h}{4\pi}\]

\[(\Delta p) \geq \frac{h}{4\pi(\Delta x)}\]

\[(\Delta p) \geq \frac{(6.626 \times 10^{-34} Js)}{4\pi(5 \times 10^{-11} m)}\]

\[\Delta p \geq 1 \times 10^{-24} \text{ kg m s}^{-1}\]

We assume the mass of the electron to be exact ($m = 9.109 \times 10^{-31}$ kg), so the uncertainty is only in the velocity.

\[\Delta p = m(\Delta \bar{V})\]

\[m(\Delta \bar{V}) \geq 1 \times 10^{-24} \text{ kg m s}^{-1}\]

Very large uncertainty (20% of actual speed)

\[\Delta \bar{V} \geq 1 \times 10^6 \text{ ms}^{-1}\]
• Bohr’s model conflicts with the uncertainty principle because if the electron is set within a confined orbit, you know both its momentum and position at a given moment.