PHY 452/562
Lasers and Modern Optics

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Lecture 8: Quantum Mechanics III

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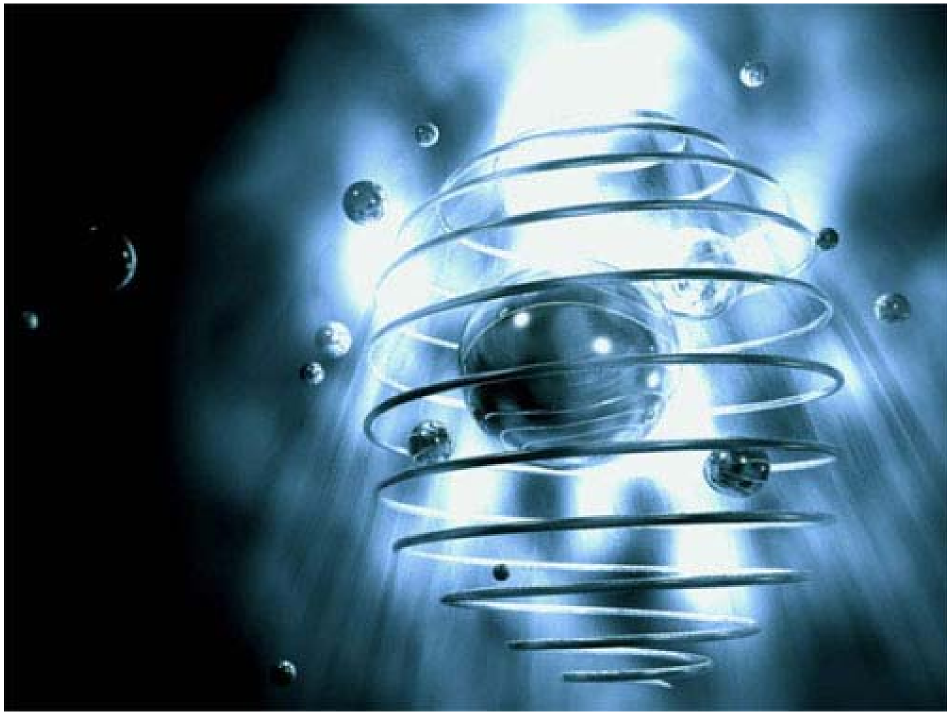
- Momentum of a photon.
- Compton effect.
- Bohr’s theory of the atom.



The momentum of a photon

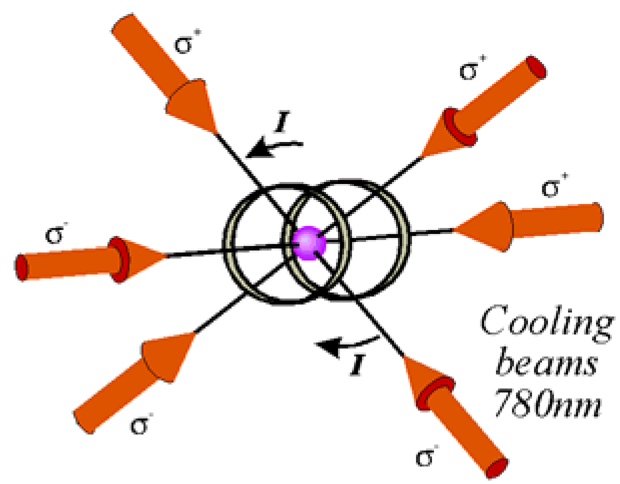
From the relativistic equation of energy-momentum:

$$E^2 = p^2c^2 + m_0^2c^4$$

when $m_0 = 0$, $E = pc = hv$

and the momentum of photon should be:

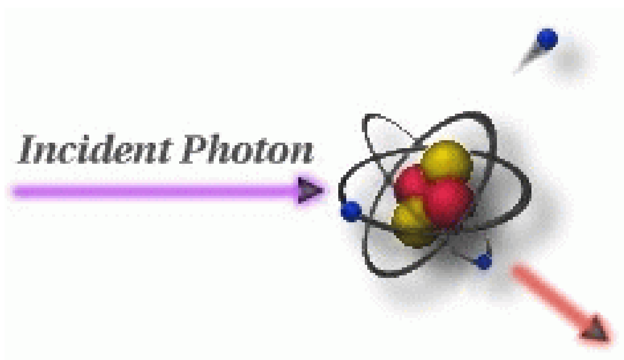
$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

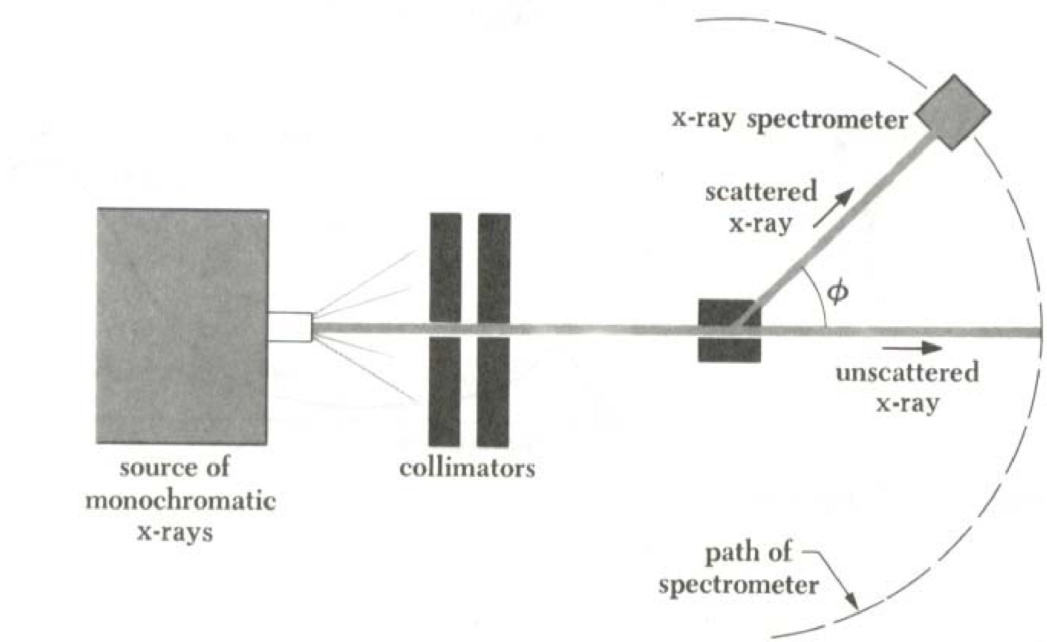


Compton effect

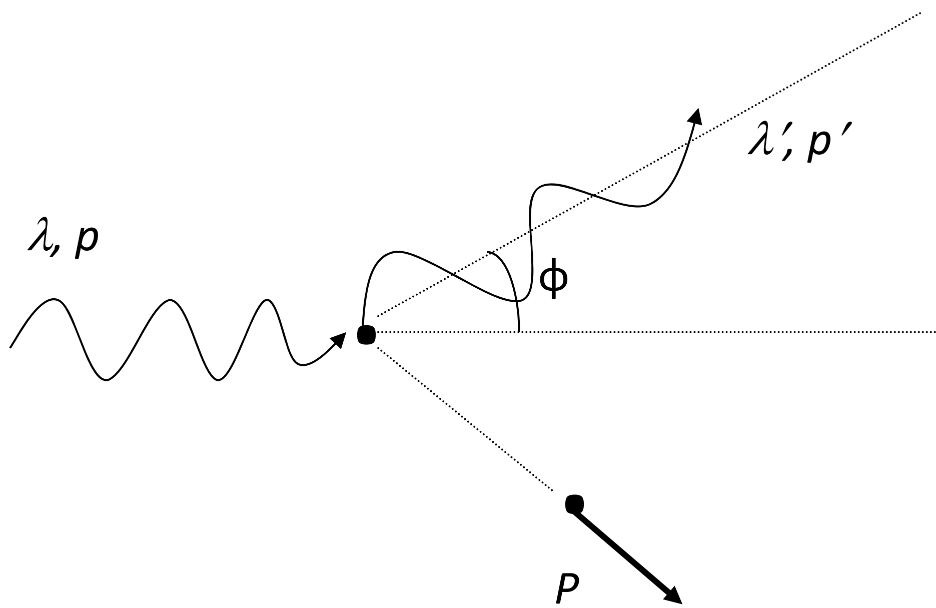
A phenomenon called Compton scattering, first observed in 1924, provides additional direct confirmation of the quantum nature of electromagnetic radiation.

When X-rays imping on matter, some of the radiation is scattered.





The electron is initially at rest with incident photon of wavelength λ and momentum *p*; scattered photon with longer wavelength λ' and momentum *p′* and recoiling electron with momentum P.



- **The key points of derivation procedure:**

(1) Relativistic theory:

$$E = h\nu = \frac{hc}{\lambda} \qquad E = mc^2$$

$$E^2 = (cp)^2 + (m_0 c^2)^2$$

When $m_0 = 0$, the momentum is equal to

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

(2) Energy conservation

$$h\nu_0 + m_0c^2 = h\nu + mc^2$$

$$\frac{h}{\lambda_0} + m_0c = \frac{h}{\lambda} + mc \qquad \textbf{(1)}$$

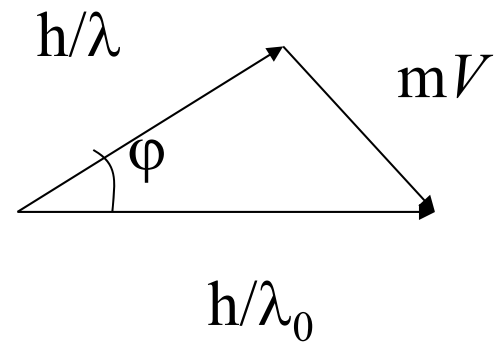
For electron, we have

$$E^2 = (cp)^2 + (m_0c^2)^2$$

$$\Rightarrow (mV)^2 = (mc)^2 - (m_0c)^2 \qquad \textbf{(2)}$$

(3) The momentum conservation

$$\frac{h}{\lambda_0}\vec{n}_0 = \frac{h}{\lambda}\vec{n} + m\vec{V}$$



$$(mV)^2 = \left(\frac{h}{\lambda_0}\right)^2 + \left(\frac{h}{\lambda}\right)^2 - 2\left(\frac{h}{\lambda_0}\right)\cdot\left(\frac{h}{\lambda}\right)\cos\varphi \quad \textbf{(3)}$$

$$(mc)^2 = (m_0c)^2 + \left(\frac{h}{\lambda_0}\right)^2 + \left(\frac{h}{\lambda}\right)^2 - 2\frac{h^2}{\lambda_0\lambda}\cos\varphi$$

(4)

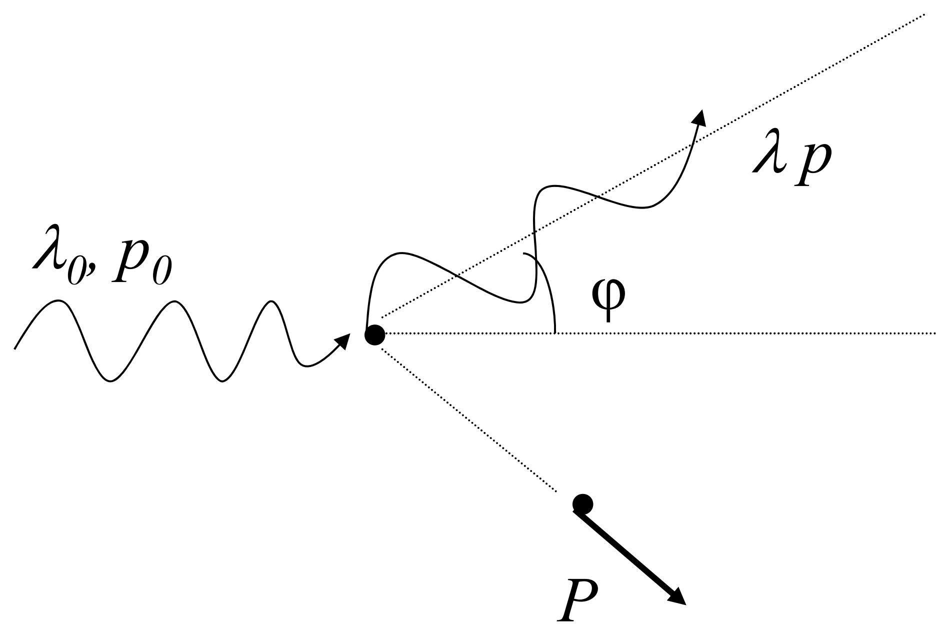
Using (1) and (4), we have

$$0 = 2m_0c\left(\frac{h}{\lambda_0} - \frac{h}{\lambda}\right) - 2\frac{h^2}{\lambda_0\lambda}(1 - \cos\varphi)$$

$$\lambda - \lambda_0 = \frac{h}{m_0 c}(1 - \cos\varphi)$$

$$= 2\frac{h}{m_0 c}\sin^2\left(\frac{\varphi}{2}\right)$$

$$= 2\lambda_c \sin^2\left(\frac{\varphi}{2}\right)$$



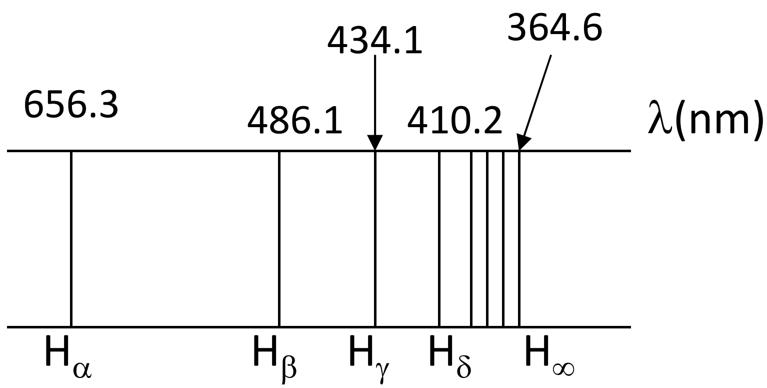
Line spectra and Energy quantization in atoms

The quantum hypothesis also plays an important role in the understanding of atomic spectra.

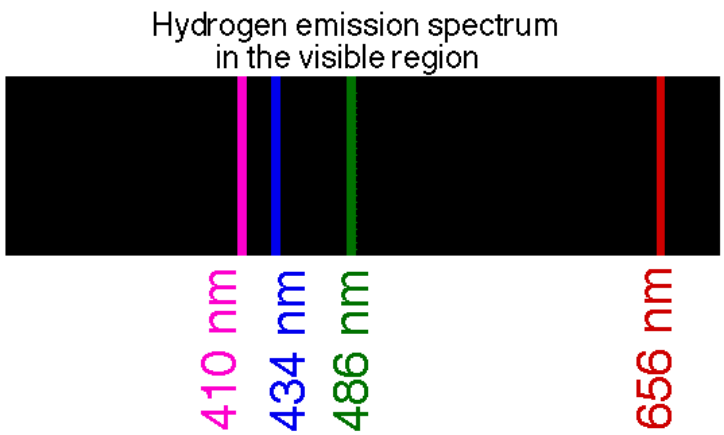
Line spectra of Hydrogen atoms

It is found that hydrogen always gives a set of spectra lines in the same position.

In 1885, Balmer found a simple formula which provides the frequencies of a group lines emitted by atomic hydrogen.



The Balmer series of atomic hydrogen.



$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right),$$

where λ is the wavelength, *R* is a constant called the Rydberg constant, and n may have the integer values 3, 4, 5,

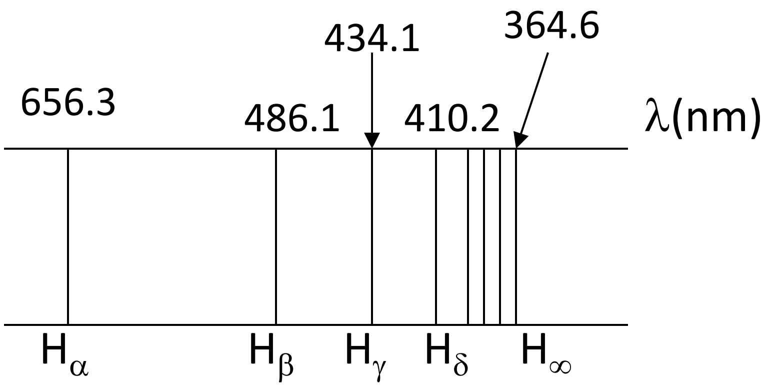
$$R = 1.097 \times 10^{7} m^{-1}$$

Substituting R and n = 3 into the Balmer formula, one obtains the wavelength:

$$\lambda = 656.3nm$$

For n = 4, one obtains the wavelength of the H_β-line. For n= ∞, one obtains the limit of the series, at λ = 364.6nm –shortest wavelength in the series.

$$\lambda = 364.6nm$$



Other series spectra for hydrogen were also discovered. These are known as Lymann, Paschen, Brackett and Pfund series.

Lymann series: $$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \qquad n = 2,3,4,\cdots$$

Paschen series: $$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \qquad n = 4,5,6,\cdots$$

Brackett series: $$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \qquad n = 4,5,6,\cdots$$

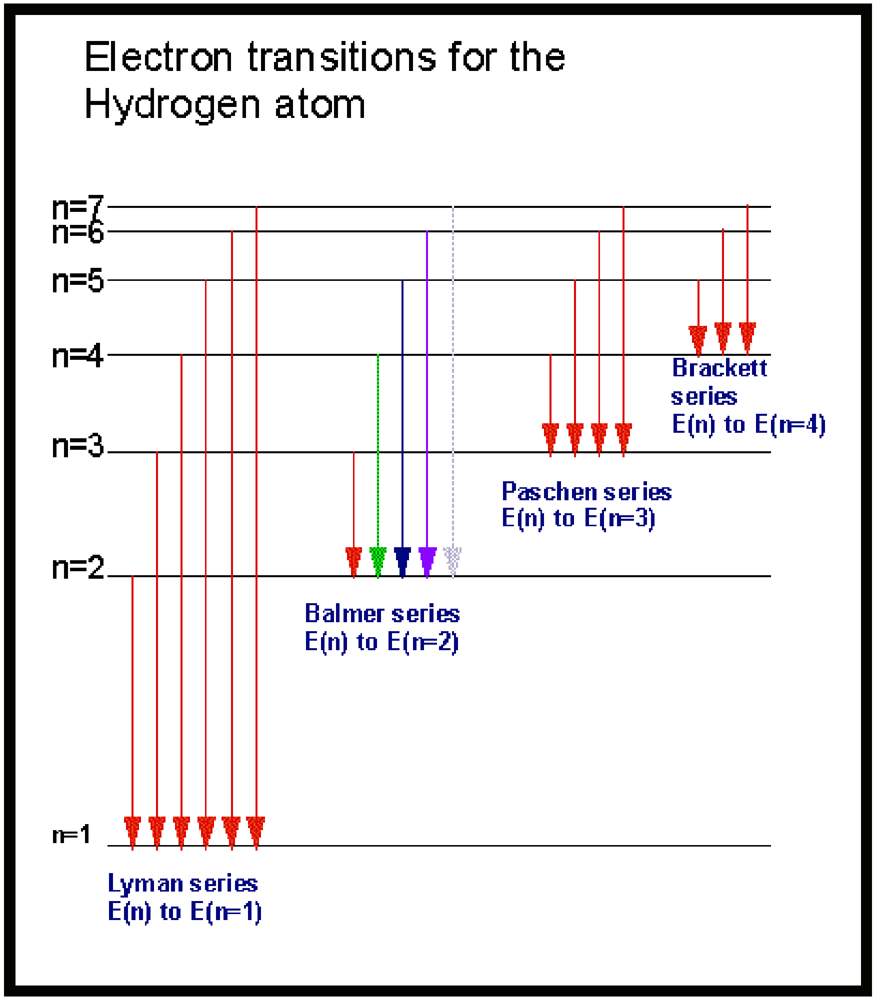
Pfund series: $$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), \qquad n = 4,5,6,\cdots$$

The Lymann series is in the ultraviolet, and the Paschen, Brackett, and Pfund series are in the infrared. All these formulas can be generalized into one formula which is called the general Balmer series.

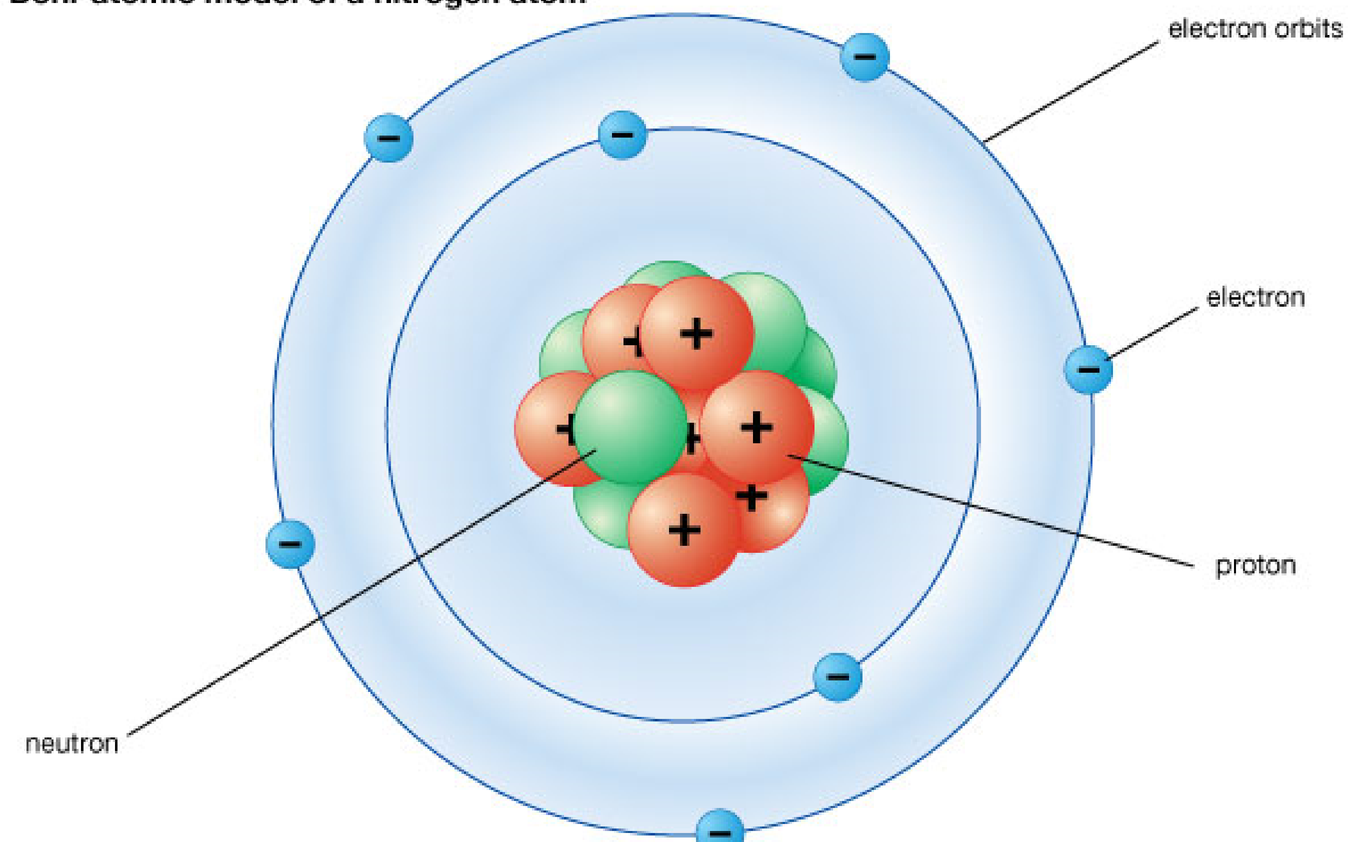
$$\frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right),$$

$$n = k+1, k+2, k+3, \cdots$$

All the spectra of atomic hydrogen can be described by this simple formula.



Bohr postulated that an electron in an atom can revolve in certain stable orbits, each having a definite associated energy, without emitting radiation.



• **The angular momentum *mvr* of the electron on the stable orbits is supposed to be equal to the integer multiple of h/2π. This condition may be stated as**

$$mvr = n\frac{h}{2\pi} \qquad n = 1,2,3,\cdots$$

where n is the quantum number, this is called the quantization condition of orbital angular momentum.

•Bohr postulated that the radiation happens only at the transition of an electron from one stable state to another. The energy of the photon is equal to the difference of the energies corresponding to the two stable states.

$$h\nu = E_n - E_k \qquad \nu = \frac{E_n - E_k}{h}$$

Another equation can be obtained by the electrostatic force of attraction between two charges and Newton's law:

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r}$$

Solving the simultaneous equations we can obtain:

$$r_n = \frac{\varepsilon_0 h^2}{\pi m e^2} n^2, \qquad v_n = \frac{e^2}{2\varepsilon_0 h n} \qquad (n = 1, 2, 3, \cdots)$$

The total energy of the electron on the nth orbit is:

$$E_n = E_k + E_p$$

$$= \frac{1}{2}mv_n^2 + \frac{-e^2}{4\pi\varepsilon_0 r_n} = -\frac{me^4}{8\varepsilon_0 h^2 n^2}$$

When the electron transits from the nth orbit to kth orbit, the frequency and wavelength can be calculated as

$$\nu = \frac{E_n - E_k}{h} = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{k^2} - \frac{1}{n^2}\right) \qquad n > k$$

$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{k^2} - \frac{1}{n^2}\right) = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right)$$

where

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$$