Prof. Eden Figueroa

Lecture 5: Fabry-Perot resonators
- Experimental Intermezzo.
- Optical resonators.
- Fabry-Perot theory.
- Resonator’s stability.
- Applications.
Experimental Intermezzo
This is one of the experiments to be performed in the lab:

Assume a beam propagating in the z-direction with a Gaussian intensity profile:

\[ I(x, y) = I_0 e^{-2x^2/w_x^2} e^{-2y^2/w_y^2} \]
The total power in the beam is:

\[ P_{TOT} = I_0 \int_{-\infty}^{\infty} e^{-2x^2/w_x^2} \, dx \int_{-\infty}^{\infty} e^{-2y^2/w_y^2} \, dy = \frac{\pi}{2} I_0 w_x w_y \]

Consider the knife edge being translated in the x-direction

\[ P(X) = P_{TOT} - I_0 \int_{-\infty}^{X} e^{-2x^2/w_x^2} \, dx \int_{-\infty}^{\infty} e^{-2y^2/w_y^2} \, dy \]

\[ = \frac{P_{TOT}}{2} - \sqrt{\frac{\pi}{2}} I_0 w_y \int_{0}^{X} e^{-2x^2/w_x^2} \, dx \]
Making the substitutions:

\[ u^2 = \frac{2x^2}{w_x^2} \quad \text{and} \quad dx = \frac{w_x}{\sqrt{2}} du \]

\[
P(X) = \frac{P_{\text{TOT}}}{2} - \sqrt{\frac{\pi}{2}} I_0 w_y \int_0^{\sqrt{\frac{2x}{w_x}}} e^{-u^2} \frac{w_x}{\sqrt{2}} du
\]

\[
= \frac{P_{\text{TOT}}}{2} - \frac{\pi}{4} I_0 w_y w_x \int_0^{\sqrt{\frac{2x}{w_x}}} e^{-u^2} du
\]

Here we use the definition of the error function:

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du
\]
To fit the data, we have to use a fitting function of the form:

\[ P(X) = \frac{P_{TOT}}{2} \left[ 1 - \text{erf} \left( \frac{\sqrt{2}X}{w_x} \right) \right] \]

\[ P_{measured} = \frac{P1}{2} \left[ 1 \pm \text{erf} \left( \frac{\sqrt{2}(X - P2)}{P3} \right) \right] \]
Fabry-Perot resonators
The first interferometer was constructed by Fabry and his colleague, Alfred Perot, in 1897.

It consisted of two perfectly flat glass plates coated on their parallel facing surfaces with thin silver films.
Jamin’s interferometer - 1856
Mach-Zehnder interferometer - 1892
Michelson interferometer - 1882
The multi-beam interferometer by A. Fabry and Ch. Perot, 1897
**Fabry-Perot Interferometers**

- A cavity with highly reflective mirrors parallel to each other
  Acts like a resonator
- Also called etalon

![Diagram of Fabry-Perot Interferometer](image)
Fabry-Perot interferometers

Resonance condition: reflected wave = 0

⇔ all reflected waves interfere destructively

\[ L = \frac{m\lambda}{2n} \]

wavelength in free space
refractive index
Relation between $r$, $r'$ and $t$, $t'$ (Stokes relations)

Forwards in time

Backwards in time

Backwards in time (showing all possible fields)

$t'tE + r^2E = E$

$r'tE + trE = 0$

$\Rightarrow t't + r^2 = 1$ and $r = -r'$

(Stokes Relations)
Calculation of the reflected wave

\[ \delta = 2\pi \frac{nL}{\lambda} \]
Calculation of the reflected wave

\[ a_{\text{reflected}} = a \left( r + t t' r' e^{i2\delta} \left( 1 + r'^2 e^{i2\delta} + r'^4 e^{i4\delta} + \cdots \right) \right) \]

\[ = a \left( r + t t' r' e^{i2\delta} \frac{1}{1 - r'^2 e^{i2\delta}} \right) \]

Use Stokes relationships

\[ r' = -r \]
\[ r^2 + t t' = 1 \]

\[ a_{\text{reflected}} = a \frac{r (1 - e^{i2\delta})}{1 - r^2 e^{i2\delta}} \]
Transmission & reflection coefficients

\[ a_{\text{reflected}} = a \frac{1 - \mathrm{e}^{i2\delta}}{1 - r^2 \mathrm{e}^{i2\delta}} r \]

\[ a_{\text{transmitted}} = a \frac{tt'}{1 - r^2 \mathrm{e}^{i2\delta}} \]

\[ R \equiv |r|^2 \]

\[
\left( \begin{array}{c}
\text{reflection} \\
\text{coefficient}
\end{array} \right) = \left| \frac{a_{\text{reflected}}}{a} \right|^2 = \frac{4R \sin^2 \delta}{(1 - R)^2 + 4R \sin^2 \delta}
\]

\[
\left( \begin{array}{c}
\text{transmission} \\
\text{coefficient}
\end{array} \right) = \left| \frac{a_{\text{transmitted}}}{a} \right|^2 = \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2 \delta}
\]