PHY 452/562
Lasers and Modern Optics

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Lecture 4: Laser beam propagation II

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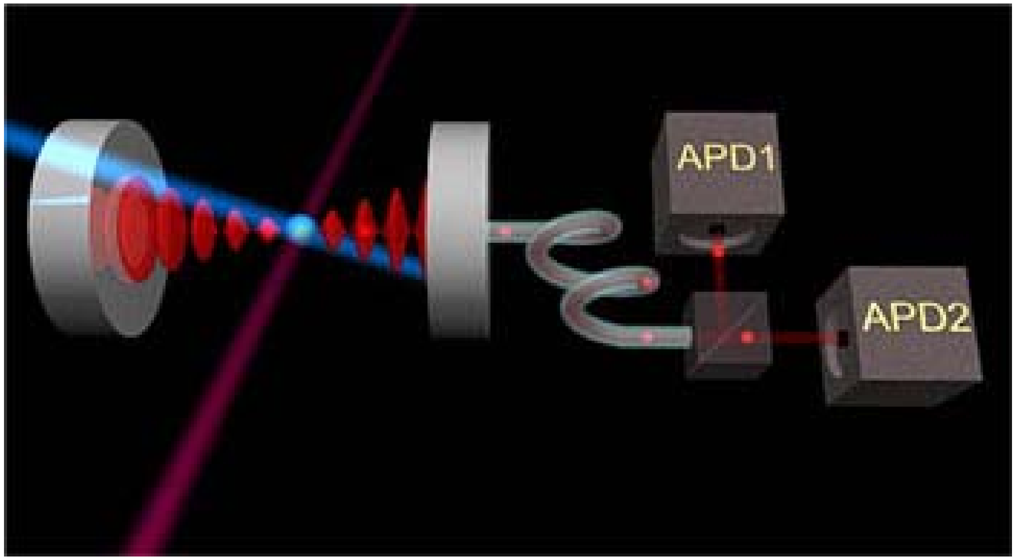
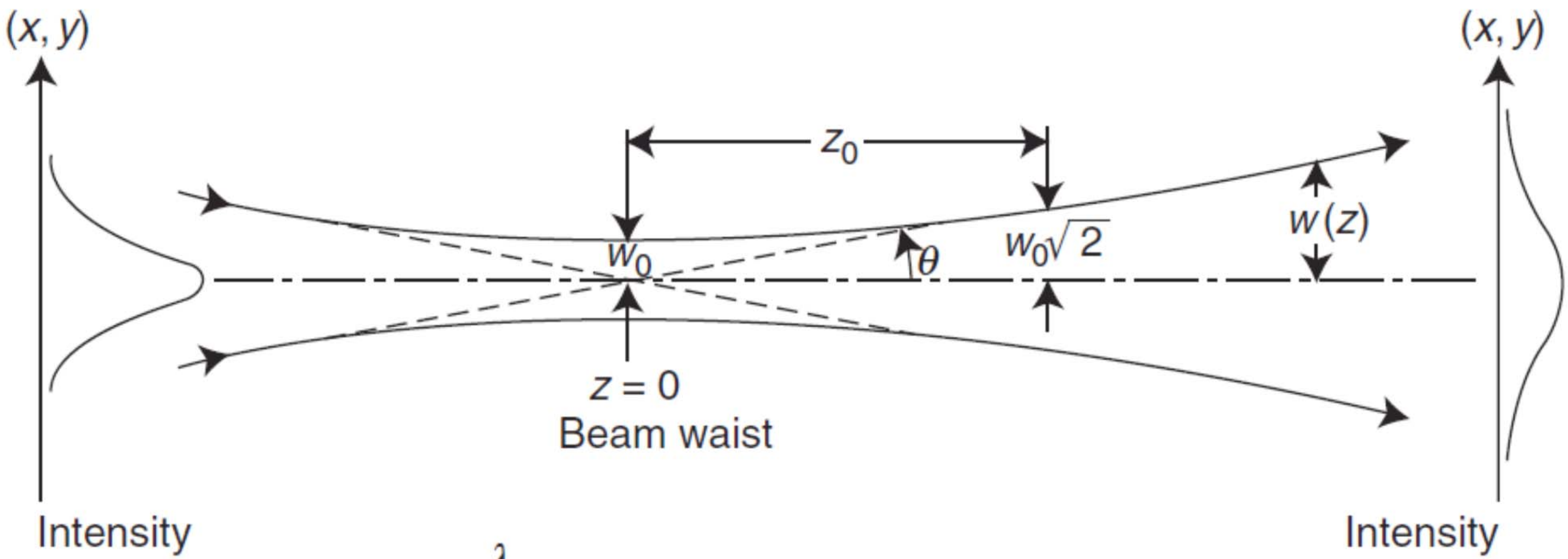
- A few problems to solve
- Propagation of Gaussian beams through optical elements
- ABCD matrix analysis for Gaussian beams.
- Higher order Gaussian solutions.

Nature 425, 268 (2003)





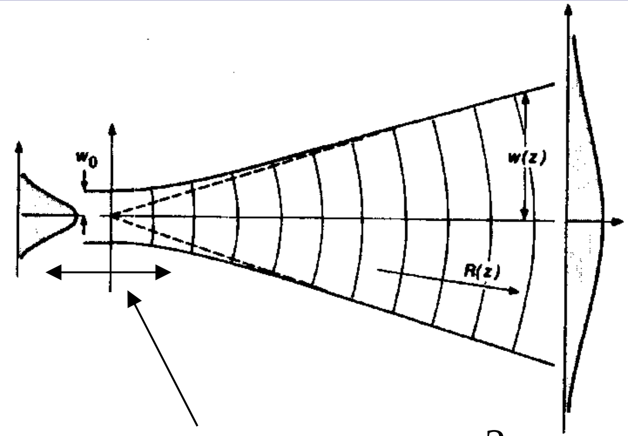
$$\theta = \frac{\lambda}{\pi w_0}$$

$$w(z) = w_0\sqrt{1 + z^2/z_0^2},\ z_0 = \frac{\pi w_0^2}{\lambda}$$

The beam has a waist at $z = 0$, where the spot size is w_0. It then expands to $w = w(z)$ with distance z away from the laser.

The beam radius of curvature, $R(z)$, also increases with distance far away.

Twice the Rayleigh range is the distance over which the beam remains about the same size, that is, remains *"collimated."*

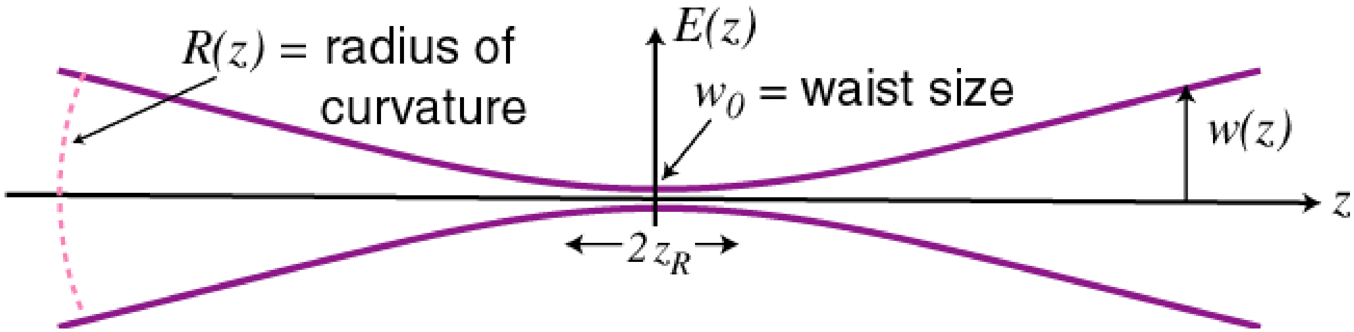


$$2z_R = 2\pi w_0^2 / \lambda$$

Waist spot size w_0	Collimation Distance $\lambda = 10.6\ \mu m$	Collimation Distance $\lambda = 0.633\ \mu m$
.225 cm	0.003 km	0.045 km
2.25 cm	0.3 km	5 km
22.5 cm	30 km	500 km

Longer wavelengths expand faster than shorter ones.

Tightly focused laser beams expand faster.



$$E(x,y,z) \propto \exp\left[-i\frac{\pi}{\lambda}\frac{x^2+y^2}{R(z)}\right]\exp\left[-\frac{x^2+y^2}{w^2(z)}\right]$$

where:

$$\frac{1}{q(z)} \equiv \frac{1}{R(z)} - i\frac{\lambda}{\pi w^2(z)}$$

← q completely determines the Gaussian beam.

$$R(z) = z + \frac{z_0^2}{z} \qquad w(z) = w_0\sqrt{1 + z^2/z_0^2} \qquad z_0 = \frac{\pi w_0^2}{\lambda}$$

Does $q(z) = q_0 + z$? This is equivalent to: $1/q(z) = 1/(q_0 + z)$.

LHS: $$\frac{1}{q(z)} \equiv \frac{1}{R(z)} - i\frac{\lambda}{\pi w^2(z)}$$ $z_R = z_0$

$$\frac{1}{q(z)} \equiv \frac{1}{z + z_R^2/z} - i\frac{1}{z_R(1 + z^2/z_R^2)} = \frac{1}{z + z_R^2/z} - i\frac{1}{z_R + z^2/z_R}$$

Now: $q(0) = i\dfrac{\pi w_0^2}{\lambda} = i\,z_R$ so $q(0)+z=iz_R+z$

RHS: $$\frac{1}{q(0)+z} = \frac{1}{z+iz_R} = \frac{z-iz_R}{z^2+z_R^2} = \frac{z}{z^2+z_R^2} - \frac{iz_R}{z^2+z_R^2}$$

$$= \frac{1}{z+z_R^2/z} - i\frac{1}{z^2/z_R+z_R}$$ which is just this.

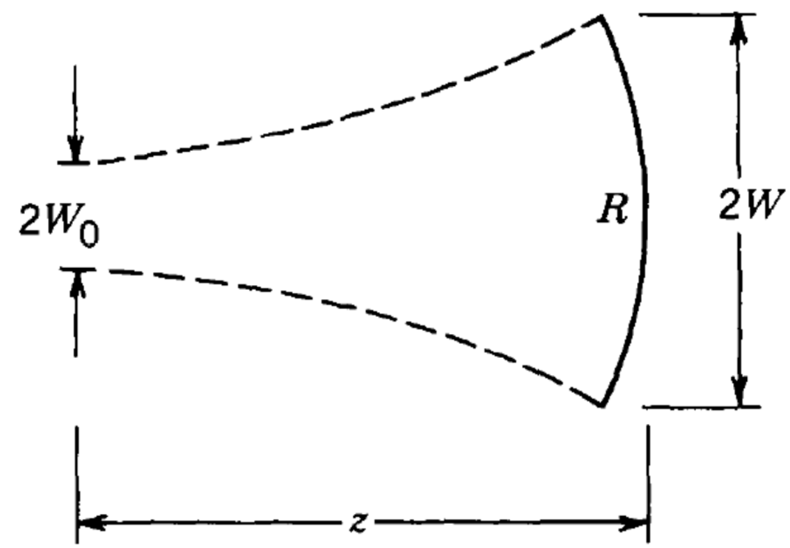
$$= \frac{1}{q(z)}$$

$$q(z) = q(0)+z$$

Assuming that the width ***W*** and the radius of curvature ***R*** of a Gaussian beam are known at some point of the beam axis, the beam waist is located at a distance:

$$z = \frac{R}{1 + \left(\lambda R/\pi W^2\right)^2}$$

$$W_0 = \frac{W}{\left[1 + \left(\pi W^2/\lambda R\right)^2\right]^{1/2}}$$

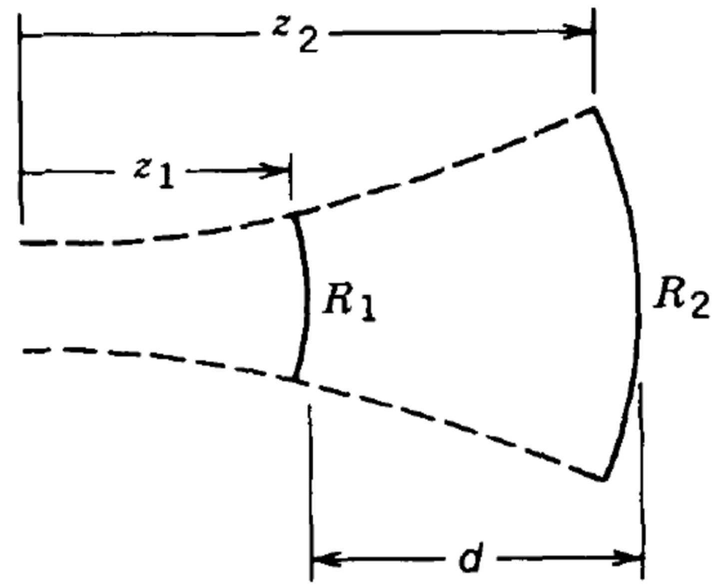


A Gaussian beam has radii of curvature R1, and R2, at two points on the beam axis separated by a distance **d**

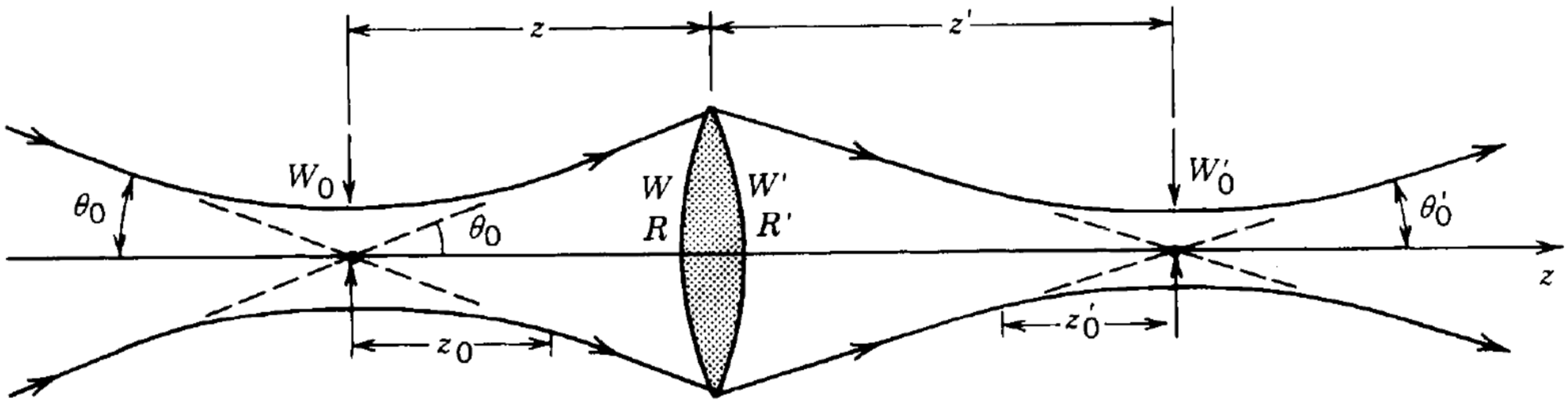
$$z_1 = \frac{-d(R_2 - d)}{R_2 - R_1 - 2d}$$

$$z_0^2 = \frac{-d(R_1 + d)(R_2 - d)(R_2 - R_1 - d)}{(R_2 - R_1 - 2d)^2}$$

$$W_0 = \left(\frac{\lambda z_0}{\pi}\right)^{1/2}$$



Transmission of a Gaussian beam through a thin lens.



$$\frac{1}{R'} = \frac{1}{R} - \frac{1}{f}.$$

$$W_0' = \frac{W}{\left[1 + \left(\pi W^2/\lambda R'\right)^2\right]^{1/2}} \qquad -z' = \frac{R'}{1 + \left(\lambda R'/\pi W^2\right)^2}$$