PHY 452/562 Lasers and Modern Optics (due Tuesday Sept. 24th 2013)

Homework # 1

Problem 1. Given the paraxial wave equation:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+2ik\frac{\partial}{\partial z}\right)\mathcal{E}_0(\mathbf{r})\approx 0$$

And a proposed solution of the form:

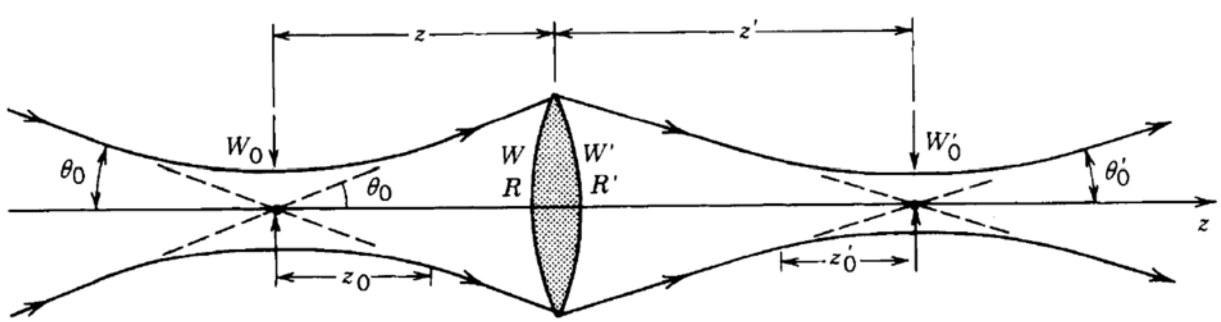
$$\mathcal{E}_0(\mathbf{r}) = Ae^{ik(x^2+y^2)/2q(z)}e^{ip(z)}$$

Prove the following:

$$\frac{\partial \mathcal{E}_0}{\partial z} = iA\left[\frac{dp}{dz}-\frac{k}{2}(x^2+y^2)\frac{1}{q^2}\frac{dq}{dz}\right]e^{ik(x^2+y^2)/2q(z)}e^{ip(z)}$$

$$\nabla_T^2\mathcal{E}_0 = A\left[\frac{2ik}{q}-\frac{k^2}{q^2}(x^2+y^2)\right]e^{ik(x^2+y^2)/2q(z)}e^{ip(z)}$$

Problem 2. A Gaussian beam is transmitted through a thin lens of focal length f (see diagram below).



(a) Show that the locations of the waists of the incident and transmitted beams, z and z', are related by:

$$\frac{z'}{f}-1=\frac{z/f-1}{(z/f-1)^2+(z_0/f)^2}$$

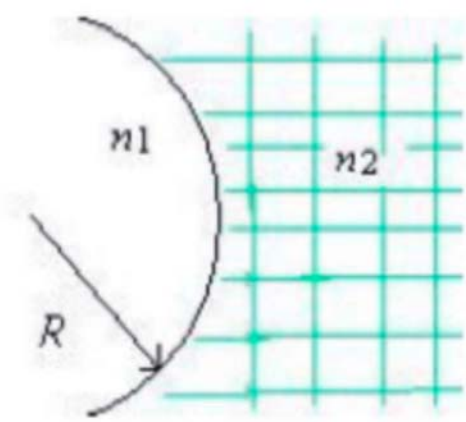
(b) The beam is collimated by making the location of the new waist z' as distant as possible from the lens. This is achieved by using the smallest ratio z_0/f (short depth of focus and long focal length). For a given ratio z_0/f, show that the optimal value of z for collimation is 2 = f + z_0.

(c) If λ = 1 μm, z_0 = 1 cm and f = 50 cm, determine the optimal value of z for collimation, and the corresponding magnification M, distance z', and width W_0' of the collimated beam.

Problem 3. Demonstrate that if a ray passes a spherical dielectric interface, the ABCD matrix describing the propagation is given by:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{n_2 - n_1}{n_2 R} & \dfrac{n_1}{n_2} \end{bmatrix}$$

where R > 0 if the surface is concave and R <0 if the surface is convex.



Additionally, solve problems 7.1, 7.9, 7.10 and 7.14 from the Milonni book (2nd edition).